Lies My Physics Text Told Me

My grade 11 physics text told me that middle A on the piano is tuned to a frequency of 440 Hz. This is true. Four hundred and forty hertz is the international orchestral standard tuning for middle A. The text also told me that the next A above middle A makes an interval of one octave, that is, eight notes. The frequency doubles every time you go up an octave. This is also true.

What frequency would a piano tuner use for the A one octave above middle A? If you said 880 Hz, you would agree with my grade 11 physics text. You would also be quite wrong. If you tune a piano using a doubling of frequency for each octave, it is unplayable. How do I know this?

After signing the contract for my first paid position as a physics teacher, I decided to invest part of my salary in the purchase of a piano, a 1906 Mendelssohn upright grand. It was in poor condition, and I didn't have much cash, so I purchased some parts from a piano supply shop, and reconditioned the instrument, replacing the rather rusty strings, fixing broken hammers, and updating the old, yellowed ivory on the keys with bright, white imitation-ivory plastic.

When it came time to tune the piano, I smugly relied on my B. Sc. in physics to develop the procedure. I purchased a tuning hammer, borrowed a signal generator from my lab at the school, and used my Hewlett Packard scientific calculator to make up a table of frequencies using 440 Hz for middle A as a starting point. The scale used in western music is the equally-tempered scale. Each octave consists of twelve semi-tones, so the ratio of the frequencies for any two consecutive semitones is the twelfth root of two. I checked the middle few octaves with the table in my physics text, and the frequencies matched. Forging ahead, I adjusted the signal generator for each key in turn, and tuned it to the calculated frequency.

Eighty-eight keys and three hours of careful tuning later, I sat down to try out my handiwork. Mouth watering, ears twitching, I poised fingers over the keyboard in a basic C chord pattern and played. The result was atrocious: atonal, sour, and thoroughly unpleasant. The piano was unplayable.

What had gone wrong? I swallowed my pride in my hard-won university degree, and slinked off to the local library, where I found a text on piano tuning. What I discovered amazed me: more lies from my physics text!

Let's return to the 440 Hz middle A. According to my physics text, the second harmonic is 880 Hz, the third is 1320 Hz, and so on. Lies! The second harmonic is more than 880 Hz, and furthermore, varies from one piano to another.

A vibrating string possesses a property called stiffness. One can see this in a plastic ruler. Take a 30 cm ruler, and it is fairly easy to bend. Cut it in half, and each half is much more difficult to bend. Cut a piece in half again, and it is almost impossible to bend. This property is called stiffness.

When a stretched string vibrates, the fundamental frequency, 440 Hz in this case, results from a standing wave with a single loop. The second harmonic arises from two loops in the standing wave. However, the stiffness of the half-string forming each loop is higher than the stiffness of the entire string. The frequency of the second harmonic is more than twice the fundamental.

When tuning a piano, the A above middle A must be tuned to the first harmonic of the A below it, not to twice the fundamental. Otherwise, it will cause a dissonant beat when the two notes are played simultaneously, as in a chord. This correction to the tuning is sometimes known to piano tuners as "octave stretch". Since it varies from piano to piano, each piano must be tuned in a custom manner. Old-fashioned piano tuners relied on a keen sense of hearing coupled with a number of rules of thumb regarding beat frequencies to produce a pleasing compromise in the tuning of the strings. Younger piano tuners rely on electronic tuners as tuning aids. These electronic tuners can be programmed to employ different octave stretches for different makes and models of pianos.

Armed with this revelation, I retuned my piano using my ears rather than a signal generator. It sounded much better.

References for further information:

Piano Servicing, Tuning, and Rebuilding, Arthur A. Reblitz, The Vestal Press, Vestal, NY.
http://members.aol.com/chang8828/contents.htm, an excellent online guide to the fundamentals of piano practice.